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Fractal flow of inhomogeneous fluids over smooth inclined surfaces and determination of their fractal dimensions and universality classes

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Abstract
Patterns formed by the flow of an inhomogeneous fluid (suspension) over a smooth inclined surface were studied. It was observed that fractal patterns are formed. There exists a threshold angle for the inclination above which global fractal patterns are formed. This angle depends on the particle size of the suspension. We observed that there are two fractal dimensions for these patterns, depending on the area from which the pattern is extracted. If the pattern is taken from the top which only consists of the beginning steps of the pattern forming, one finds two fractal dimensions, i.e. 1.35–1.45 and 1.6–1.7, in which the first one is dominant. And if the entire pattern is taken, then fractal dimension 1.6–1.7 is observed. The first fractal dimension belongs to the class of flow of water over an inhomogeneous surface, and the second one corresponds to the river network. This may imply that both universality classes are present. However, here disorder is present in the fluid and is transferred to the surface. We have also determined the fractal dimension of the patterns formed below the threshold angle. Testing Horton's laws on different patterns does not lead to a conclusive result. More investigations may give a reliable conclusion.

1. Introduction

The flow of fluid in a random medium as an instance of collective nonlinear transport with strong disorder [1–3] has attracted much attention. Macroscopic transport occurs only when the driving force exceeds a threshold magnitude. Near the threshold, there is some evidence of critical behaviour, including a diverging correlation length. Examples of this behaviour can be seen in the patterns formed on the window pane during rain [1]. In all these examples, the environment is strongly disordered but through this environment, a simple body flows. In contrast with this example, the system concerning us is a system in which the surface is clean and the fluids do not erode it, but disorder is present within the moving fluid.

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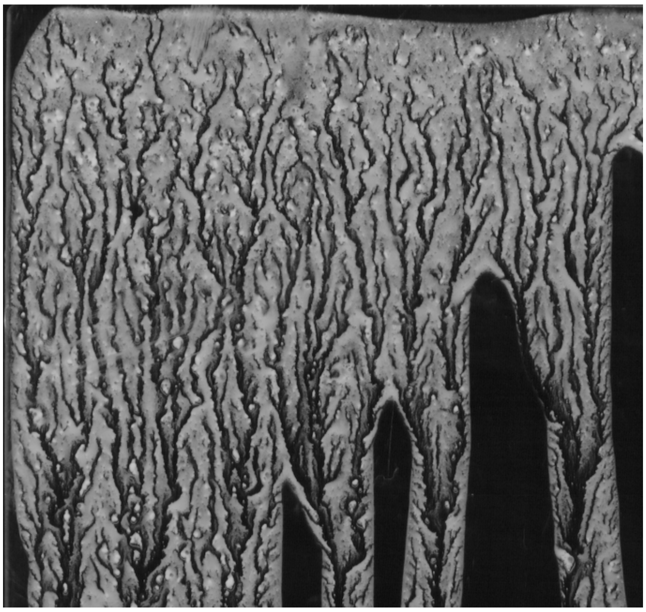

Figure 1. The digitized pattern of the flow of the yogurt–water suspension over an inclined surface. The white areas are precipitated yogurt, the black areas are washed by the flow of water. This is above the threshold angle at which we can see many global fractal trees.

2. Experimental procedures

The experiments were carried out with three suspensions: yoghurt and water, talcum powder and water, and flour and water, in controlled proportions. The top surface of the plate, on which the fluid is poured, is covered by a clean plate of glass. The ratio of mixture was kept fixed during all experiments. The volume ratio of yogurt to water (for example) was 100 to 150. Once the fluid is poured down over an inclined surface, the precipitation patterns, left by the fluid, have obvious fractal features. These patterns were registered and digitized by applying a scanner and a computer.

We observe that there exists a critical angle of inclination. If the angle of the inclined plane is below this threshold, the fractal pattern is not global. We obtained this angle to be in the range (10°–12°) for yogurt, (8°–10°) for talcum powder, and (34°–36°) for flour. The threshold angle is where one observes the first global fractal. Here we report the scaling properties of the system. Our observations are consistent with the theoretical results on the flow of fluid over rough surfaces [4, 5] at the beginning of pattern formation, i.e. on the top of the plate, also consistent with the river network for the whole pattern. Figure 1 shows pattern formation above the threshold angle and figure 2 shows pattern formation below the threshold angle. A frequency distribution of the particle size of yoghurt is shown in figure 3. Specifications of these suspensions obtained through measurements by a particle size analyser are given in table 1.

3. Image analysis

After importing a digitized pattern, the fractal dimension is measured employing various methods. For calculating the fractal dimension we employed two different methods.

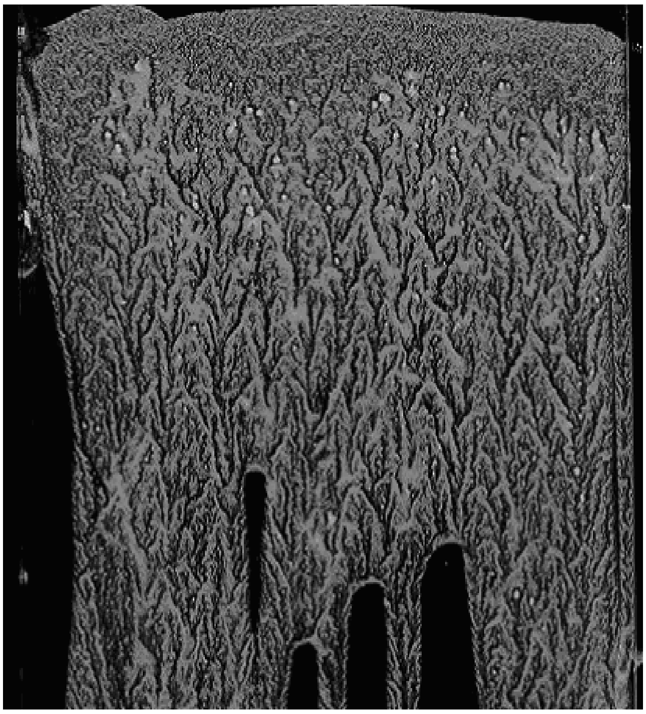

Figure 2. Digitized pattern formation below the threshold angle. There is no global fractal tree connecting the top to the bottom.

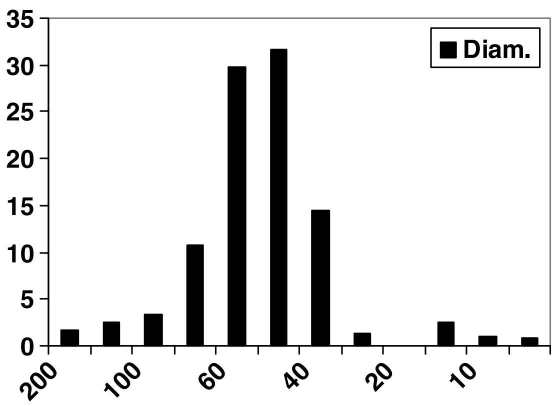

Figure 3. The frequency distribution of particle diameters for yoghurt.

Method 1. The mass of a particular stream, m, is related to its downward length of flows [1]:

$$m(\ell) = \ell^{d_f}.$$

Here the mass refers to the total surface area of a stream and ℓ is its length. In this method several patterns are taken from a fractal tree at different scales and for each of them $m(\ell)$ and ℓ are calculated and plotted. The exponent d_f is then calculated. The average of all curves yields the final exponent d_f. Figure 4 shows one of these graphs for yogurt and its slope. In

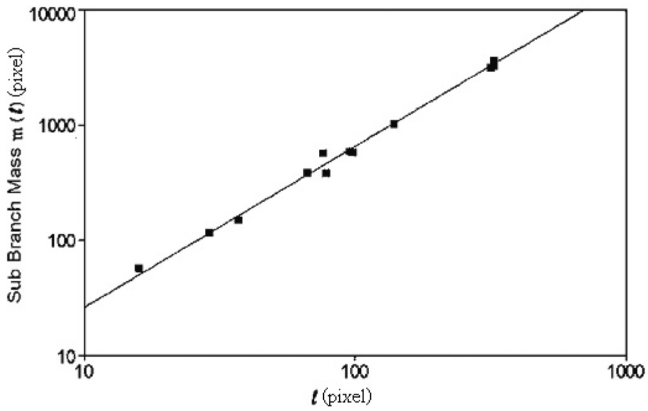

Figure 4. The fractal dimension of the yogurt mix poured on an inclined surface; $m(\ell)$ is the surface area of the sub-branch and ℓ its length scale.

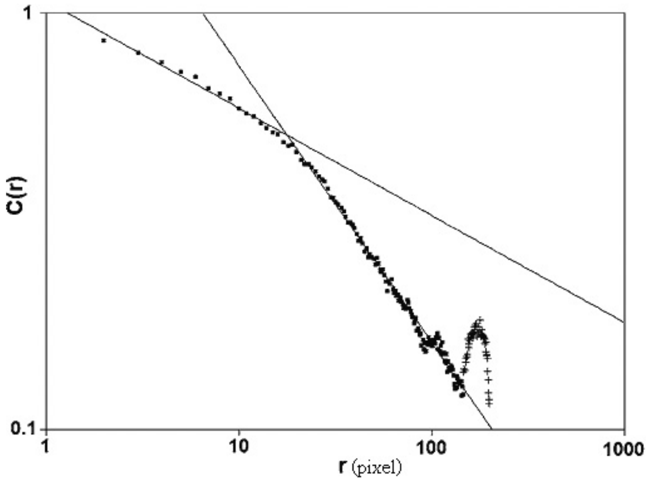

Figure 5. The correlation function for the downward direction of yogurt, for the top portion of the pattern. For the mid-section $\alpha = 0.6$, while for the beginning section $\alpha = 0.3$. This corresponds to fractal dimensions of 1.4 and 1.7, respectively.

Table 1. Particle specifications.

Suspension	Median diameter (μm)	Modal diameter (μm)	Surface area ($m^2 g^{-1}$)
Yoghurt	49.37	48.28	1.14
Flour mix	17.75	12.29	0.28
Talcum mix	24.75	24.4	0.22

this method d_f is obtained as being in the range 1.35–1.45 for the top of the pattern.

Method 2. The second method which is studied is the correlation function method [1, 6], which is defined as follows:

$$C(r) \equiv \frac{1}{N} \sum_i \rho(\vec{r})\rho(\vec{r}_c + \vec{r}). \quad (1)$$

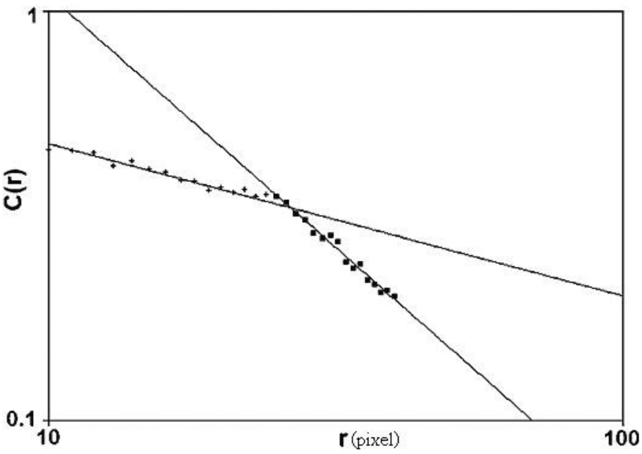

Figure 6. The correlation function for the flour chosen from the top of the pattern. The fractal dimension for the mid-section is 1.34 and for the beginning section it is 1.7.

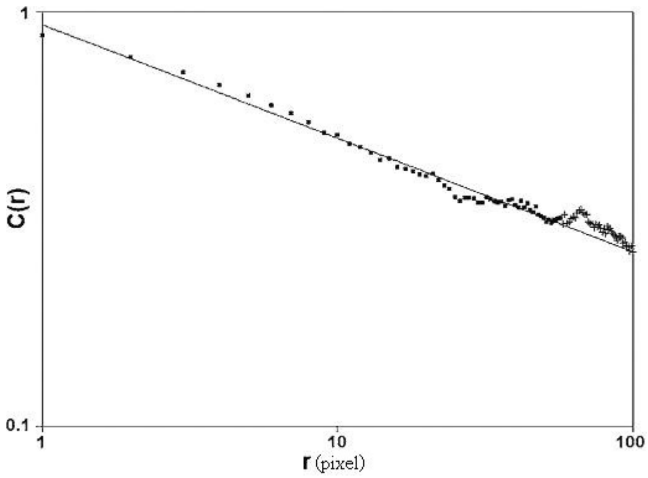

Figure 7. The correlation function of the yogurt for the whole pattern. The fractal dimension is 1.7.

Here ρ is the local density; if the point of location $\vec{r}$ belongs to the structure, $\rho(r) = 1$, and if it does not belong to it, then $\rho(r) = 0$; N is the total number of points that are used for calculating the correlation function. The function $C(\vec{r})$ represents the probability that the two points belong to the pattern. In the isotropic fractal pattern we expect $C(r)$ to depend not on the direction, but only on the distance; therefore $C(\vec{r}) = C(r)$.

In the present case the horizontal and vertical directions clearly differ; therefore we expect $\vec{r}$ to have to be parallel to the downward direction. The correlation function satisfies a power law in the form

$$C(r) \sim r^{-\alpha}. \tag{2}$$

The mass of the stream $m(\ell)$ can be expressed in terms of the correlation function from which it follows that the fractal dimension is [6]

$$d_{\mathrm{f}} = 2 - \alpha. \tag{3}$$

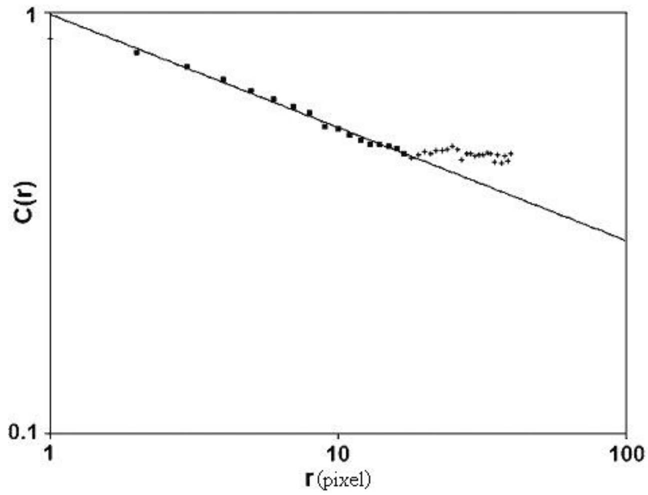

Figure 8. The correlation function for the entire pattern of the talcum powder. The fractal dimension is 1.73.

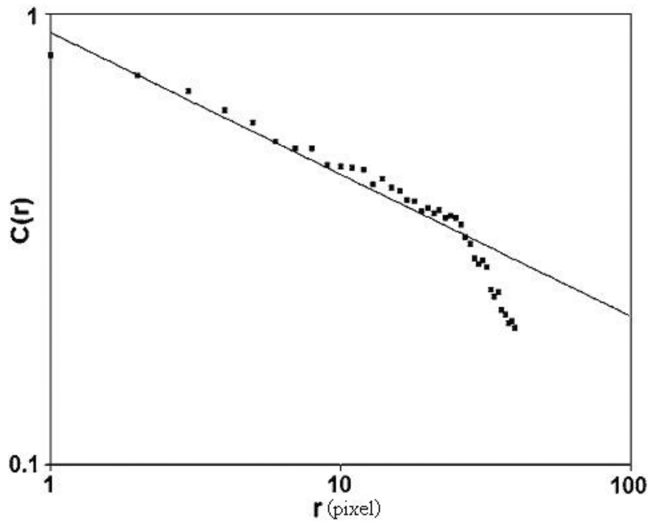

Figure 9. The correlation function of flour, all points included, giving a fractal dimension of 1.69.

This relation is the same as the one given by Vicsek, except that it is anisotropic. As noted in [1], anisotropy does not alter the scaling relation. Figure 5 plots $C(r)$ versus r for the downward flow of the yogurt suspension, in which the top part of the pattern was chosen.

The curve has two exponents in two regions, one at the beginning of the curve and the other in the mid-section. For the mid-section, $\alpha = 0.6$, while for the beginning section, $\alpha = 0.3$. This corresponds to fractal dimensions of 1.4 and 1.7, respectively. Figures 6 show the same results for flour. Figures 7–9 show the correlation functions for the yoghurt, talcum powder, and flour suspensions, respectively, when the whole pattern was chosen for analysis. The fractal dimension for all of them lies in the range 1.6–1.75.

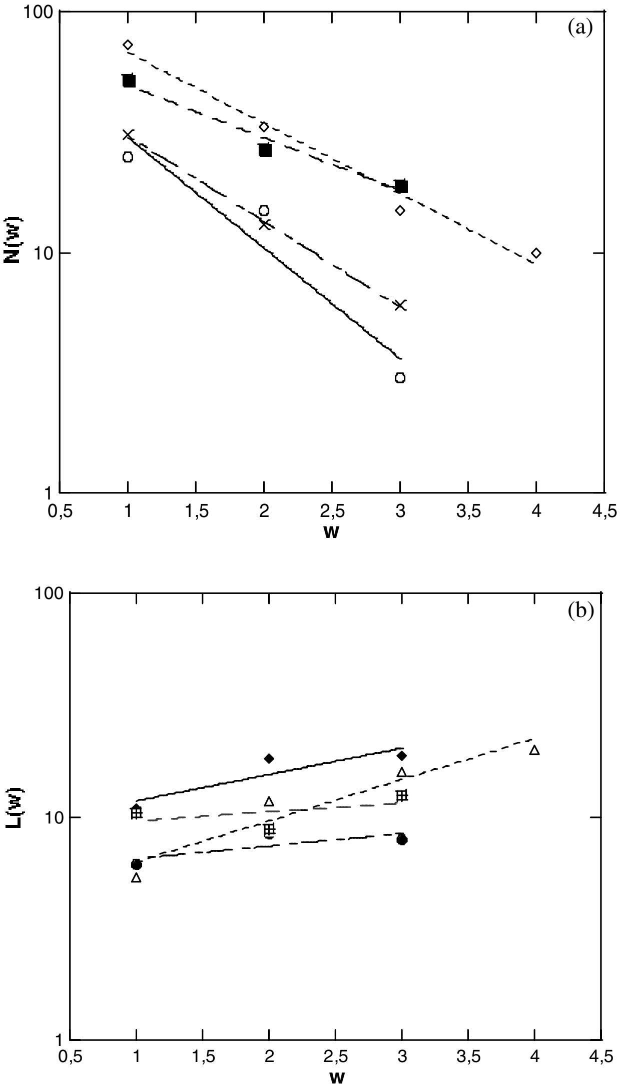

Figure 10. Horton analyses. (a) $N(w)$ and (b) $L(w)$ versus w for four streams. As we can see from the data, they are not conclusive.

Horton's analysis

According to Horton–Strahler stream ordering [7, 8], when a stream segment of order w_1 merges with a stream segment of order w_2, the outgoing stream will have the order of w, given by

$$w = \max(w_1, w_2) + \delta_{w_1,w_2} \tag{4}$$

where δ is the Kronecker delta [7]. The number of streams of order w is N_w, while L_w is the average length of streams of order w. Horton's laws state that the bifurcation ratio R_B and the length ratio R_L, given by $R_w = N_w/N_{w+1}$ and $R_L = L_{w+1}/L_w$, are constant, or independent of w, and the fractal dimension of the river $D_f \approx \log R_B/\log R_L$ [8]. We tried to test Horton's

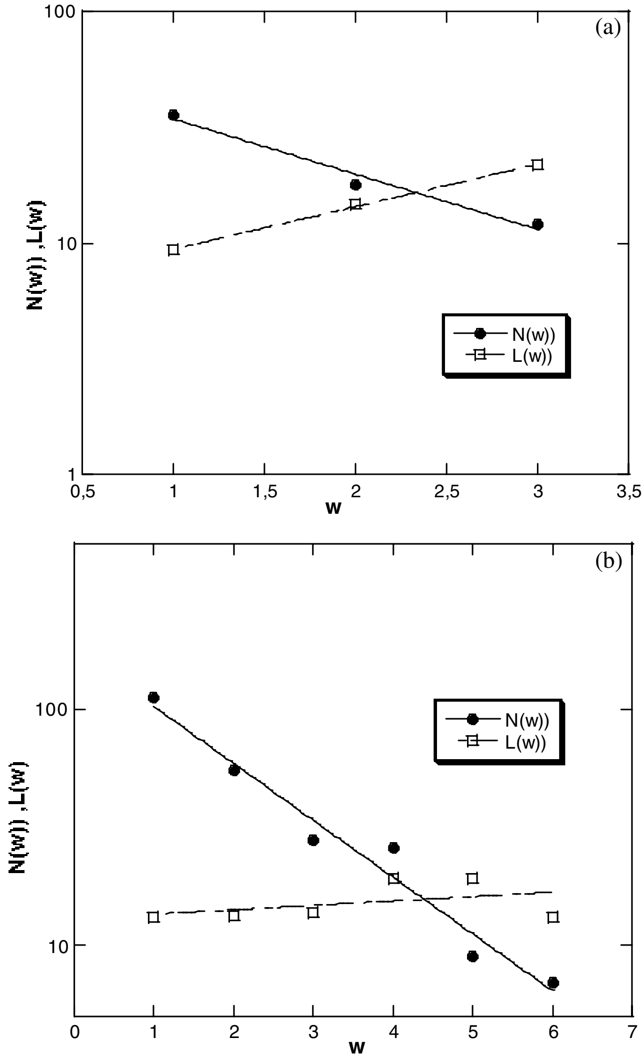

Figure 11. Horton analyses. (a) A single channel; (b) the whole pattern including linked channels.

laws on our patterns; we chose the patterns randomly from different suspensions, different dilutions and different angles of inclination of their surfaces. In some patterns we have single channels apart from each other, but in some others we have linked channels. We tested Horton's laws on them; the results are inconclusive as can be seen from figures 10 and 11. We suggest that to reach a reliable conclusion we must have more investigations including higher resolution in the patterns and this is our plan for the future.

4. Conclusion

Narayan and Fisher [4] proposed a model for nonlinear behaviour of the flow of fluid over a rough random surface. In this model it is assumed that the disorder is strong enough to break the flows into several channels. As the inclination angle of the surface is slowly increased, the fluid collects into lakes with more depth at the lower end of the lake. Any further increase in the inclination will cause the fluid to find a way to the adjacent lakes lower down, forming clusters of lakes. Therefore, as the inclination angle of the surface becomes larger, the length of the clusters formed increases, reaching an infinite value at a critical threshold angle. Below this limit it is possible to see many isolated clusters which are totally disconnected from the flow. At threshold, which is equivalent of the defining transition, there exists at least one flowing river from top to bottom (e.g. at least one cluster whose correlation length is infinite).

Mean field theory also predicts the value $4/3$ for the fractal dimension of such grids [4]. The system under study is different from the model of Narayan and Fisher, because the disorder is in the fluid not in the surface. in spite of this difference, the measured fractal dimension for the top of the pattern. i.e. for the early stages of the formation of the pattern, is 1.4 ± 0.05, which is in good agreement with the prediction of the model by Narayan and Fisher [4] and the mean field theory [4], and also with experimental observations in an experiment in which the movement of the rain over an inclined dirty surface has been studied [1]. In this experiment the fractal dimension was measured to be 1.37 ± 0.05, which is consistent with our measurement for the early stages of pattern formation.

On the other hand, for the whole of the pattern our measured fractal dimension is 1.6–1.75, which is in good agreement with the fractal dimension for river networks [1]. Apparently the system studied before the joining of the streams may be in the same class of universality as the model of Narayan and Fisher [4], whereas after joining the streams we may have a river network in which the fractal dimension is 1.6–1.7 [1]. Finally, we tested Horton's laws for river networks on our patterns; the results are inconclusive and more investigation is necessary to decide whether or which of the patterns are Hortonian.

Acknowledgments

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